GRADE 12 MATHEMATICS

FIRST SEMESTER FINAL EXAMINATION

Student's Name:_____

Id.No:_____

Value: 70%
Time allotted: 3 : 00**hrs**
Date: _____

General Direction
The exam booklet contains eleven **(11)** pages including the cover page. Please check that all pages are included.

Specific Directions

1. Read the questions given carefully and attempt all questions accordingly.
2. Use the attached answer sheet to give your answers.
3. Cheating on exam or attempting to help other examinee will automatically result in **"F"** grade.
4. Do not detach the exam paper.
5. Any measure taken by the invigilator won't be compromised.

Direction: Choose the correct answer for each of the following questions.

1. Which of the following defines a sequence?

   A. It is a function whose domain is the set of integers

   B. It is a function whose domain is the set of real numbers

   C. It is a function whose domain is the set of natural numbers

   D. It is a function whose domain is the set of rational numbers

2. Let $\{5, 8, 11, \ldots\}$ be a sequence. Then the nth term of the sequence a_n is given by

   A. $a_n = 2n + 3$ C. $a_n = 2n + 8$

   B. $a_n = 3n + 2$ D. $a_n = 3n - 8$

3. If $b_n = \frac{9}{2n}$ is the nth term of a sequence, then the 18-th term of the sequence is

   A. $\frac{1}{2}$ C. $\frac{3}{2}$

   B. $\frac{3}{4}$ D. $\frac{1}{4}$

4. Which of the following set of ordered pairs are points on the graph of the sequence $\{c_n\}$, where $c_n = 2 - \frac{2}{n}$?

   A. $\{(1, 0), (2, 1), (3, \frac{4}{3}), (4, \frac{3}{2})\}$

   B. $\{(1, 0), (2, 2), (3, \frac{4}{3}), (4, 3)\}$

   C. $\{(1, 1), (2, 1), (3, \frac{4}{3}), (4, 4)\}$

   D. $\{(1, 0), (2, 0), (3, 0), (4, 0)\}$

5. Arithmetic progression is a sequence in which each term except the first term is obtained by

   A. adding any number to the preceding term.

   B. multiplying the fixed number to the preceding term.

   C. dividing the fixed number to the preceding term.

   D. adding the fixed number to the preceding term.

6. Which one of the following general terms is the nth term of an Arithmetic sequence?

   A. $a_n = 2 + \frac{1}{n}$ C. $a_n = 2n^2 + 4$

   B. $a_n = \frac{1}{3}n - 6$ D. $a_n = \frac{1}{n^2}$

7. The Arithmetic mean of 78 and 100 is

   A. 89 C. 90

   B. 92 D. 88

8. A geometric progression is a sequence in which each term except the first term is obtained by

   A. multiplying any number to the preceding term.

   B. dividing any number to the preceding term.

   C. multiplying the fixed number to the preceding term.

   D. adding the fixed number to the preceding term.

9. If the first term of a Geometric progression is $G_1 = 20$ and the common ratio $r = \frac{1}{2}$, then the 10th term G_{10} is

   A. $\frac{5115}{128}$ C. $-\frac{1023}{512}$

   B. 40 D. -40

10. Which of the following is NOT the expression of $3^3 + 6^3 + 9^3 + 12^3 + 15^3 + 18^3 + 21^3 + 24^3 + 27^3 + 30^3$ in Sigma notation?

    A. $3\sum_{k=1}^{10}(3^2k^3)$ C. $3^3\sum_{k=1}^{10}(k)^3$

    B. $\sum_{k=1}^{10}(3k)^3$ D. $\sum_{k=1}^{10}(k)^3$

11. Let $a_n = \frac{1}{(n+1)(n+2)}$ be the nth a sequence. What is the nth partial sum S_n of this sequence?

    A. $S_n = \frac{1}{n} - \frac{1}{n+2}$ C. $S_n = 1 - \frac{1}{n+1}$

    B. $S_n = \frac{1}{n+1} - \frac{1}{n+2}$ D. $S_n = \frac{1}{2} - \frac{1}{n+2}$

12. The sum of the first ten terms of an arithmetic sequence $\{a_n\}$ is 60; the sixth term is 10. Then what is the general term a_n of the sequence?

    A. $a_n = 8n + 48$ C. $a_n = 8n + 38$

    B. $a_n = 8n - 48$ D. $a_n = 8n - 38$

13. If the first term a_1 and the 9th term a_9 of an Arithmetic progression are respectively 10 and 58, then the 10th partial sum S_{10} of the Arithmetic progression is

    A. 370 C. 320

    B. 740 D. 640

14. The sum of the first 6 terms of the sequence $\{\frac{2}{3}, \frac{2}{9}, \frac{2}{27}, \ldots\}$ is equal to

A. 1
C. $\frac{728}{2187}$
B. $\frac{728}{729}$
D. $\frac{728}{2187}$

15. The sum of the first four terms of a Geometric sequence is 12, and the sum from fifth to eighth term is 192. Which one of the following is the possible first three terms of the Geometric sequence?

A. $\frac{-12}{5}, \frac{24}{5}, \frac{-48}{5}$
C. $\frac{4}{5}, \frac{-8}{5}, \frac{16}{5}$
B. $\frac{-4}{5}, \frac{8}{5}, \frac{-16}{5}$
D. $\frac{12}{5}, \frac{-24}{5}, \frac{48}{5}$

16. Which one of the following is true about a series?

A. A finite series is a series with an infinite number of terms.
B. An infinite series is a series with finite number of terms.
C. $29 + 25 + 21 + \cdots$ is an infinite series.
D. $10 + 20 + 30 + \cdots$ is a finite series.

17. Which of the following is true about the convergence or divergence of a Geometric sequence?

A. The sequence $\{G_n\}$ converges to a real number M as $n \to +\infty, G_n \to +\infty$.
B. The sequence $\{G_n\}$ diverges to a positive infinity as $n \to +\infty, G_n \to +\infty$.
C. The sequence $\{G_n\}$ converges to a real number M as $n \to +\infty, G_n \to -\infty$.
D. The sequence $\{G_n\}$ diverges to a negative infinity as $n \to +\infty, G_n \to +\infty$.

18. Which of the following Geometric sequence is convergent?

A. $\{G_n\}$, where $G_n = \frac{1}{5}(5)^n$
C. $\{G_n\}$, where $G_n = \frac{8}{3}\left(\frac{3}{8}\right)^n$
B. $\{G_n\}$, where $G_n = \frac{1}{2}(-2)^n$
D. $\{G_n\}$, where $G_n = \frac{4}{7}\left(\frac{7}{4}\right)^n$

19. The Geometric series $\sum_{n=1}^{\infty} G_1 r^{n-1}$ is convergent for the case when

A. $r > 1$ or $r < -1$.
C. $r = 1$ or $r = -1$.
B. $-1 \leq r \leq 1$.
D. $-1 < r < 1$.

20. Which one of the following series is a divergent series?

A. $4 + 8 + 16 + \cdots$
C. $81 - 54 + 36 - 24 + \cdots$
B. $81 + 54 + 36 + 24 + \cdots$
D. $24 - 16 + \frac{32}{3} - \frac{64}{9} + \cdots$

21. The sum of the terms of the Geometric sequence $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$ is

A. 1
C. 2
B. $\frac{1}{2}$
D. $\frac{4}{3}$

22. The value of the series $\sum_{n=1}^{\infty} 4\left(5^{(2-n)}\right)$ is

A. 20
C. 100
B. 25
D. 16

23. An infinite geometric series representation of the repeating decimal $4.8646464\ldots$ is

A. $4.8 + 0.064 + 0.0064 + 0.000064 + \cdots$
B. $4.8 + 0.0064 + 0.00064 + 0.0000064 + \cdots$
C. $4.8 + 0.064 + 0.00064 + 0.0000064 + \cdots$
D. $4.8 + 0.64 + 0.0064 + 0.000064 + \cdots$

24. Which of the following recurring decimal converges to the corresponding fraction?

A. $0.232323\cdots = \frac{23}{100}$
C. $7.313131\cdots = \frac{31}{99}$
B. $0.5555\cdots = \frac{5}{9}$
D. $63.7777\cdots = \frac{70}{9}$

25. Let $G_n = 4(5^{(n-1)})$ be the general term of a Geometric progression. How many terms of the Geometric progression will be needed so that the sum of the sequence is $78,124$?

A. 5
C. 7
B. 6
D. 8

26. A Doctor prescribed 42 pills of medicine for his patient to be taken in the first week. Given that the patient should decrease the dosage by 7 pills every week, what is the week in which the patient stops taking the medicine completely?

A. 7th week
C. 5th week
B. 6th week
D. 4th week

27. To build up endurance, you start an exercise program in which you exercised 60 minutes for the first day, 65 minutes for the second day, 70 minutes for the third day, 75 minutes for the fourth day, and so on, each day extending your exercise by five minutes. If you continue at this rate, ending at 5 hours, what is the total time you spend exercising?

A. $8,640$ minutes
C. $8,280$ minutes
B. $8,460$ minutes
D. $8,100$ minutes

28. A pendulum that is released to swing freely travels $30m$ on the first swing. On each successive swing, the pendulum travels 75% of the distance of the previous swing. What is the total distance the pendulum swings before it comes to rest?

A. $40m$ C. $120m$

B. $90m$ D. $45m$

29. The rate of change of one quantity with respect to another quantity when their values increase at the same time is

A. positive. C. zero.

B. negative. D. undefined.

30. Which one of the following is the average rate of change of $f(x) = x^3 - 2x$ over the interval $[1, 3]$?

A. -11 C. 11

B. 10 D. -10

31. What is the instantaneous rate of change of $f(x) = x^2 + 5x$ at $x = -3$?

A. 1 C. -1

B. 0 D. 2

32. What is the slope of the secant line to the graph of $f(x) = -x^2 + 2$ over the interval $[-3, 1]$?

A. $\frac{5}{2}$ C. -2

B. 2 D. $-\frac{5}{2}$

33. Which one of the following is true about a function $y = f(x)$ and $x = a$?

A. The gradient of the line through $(a, f(a))$ and $(a + h, f(a + h))$ is $\frac{f(a+h)-f(a)}{a}$.

B. The gradient of the tangent line to f at $x = a$ is $\frac{f(x)-f(a)}{a}$.

C. The gradient of a tangent line is the average rate of change.

D. The gradient of the function f at $x = a$ is $\frac{f(x)-f(a)}{x-a}$.

34. What is the slope of the tangent line to the function $f(x) = \frac{1}{\sqrt{x}}$ at $x = 2$?

A. $\frac{-1}{2\sqrt{2}}$ C. $\frac{-1}{4\sqrt{2}}$

B. $\frac{1}{4\sqrt{2}}$ D. $\frac{1}{\sqrt{2}}$

35. The gradient of the function $f(x) = x^3 + 1$ at any point $x = a$ is

A. a^2 C. $3a^2$

B. $2a^2$ D. $4a^2$

36. Let $f(x) = kx^2 - 5$. If $f'(3) = -12$, what is the value of k?

A. -6 C. -3

B. -2 D. -4

37. If $f(x) = 3x^5 - \sqrt{2}x^3 + 10$, then what is the value of $f'(\sqrt{2})$?

A. $60 - 3\sqrt{4}$ C. $60 - 3\sqrt{2}$

B. $60 - 6\sqrt{2}$ D. 48

38. What is the derivative of $f(x) = \frac{1}{3x^{12}}$?

A. $\frac{1}{36x^{-13}}$ C. $\frac{1}{36x^{11}}$

B. $\frac{-4}{x^{11}}$ D. $\frac{-4}{x^{13}}$

39. Which one of the following is true about the derivative of a polynomial function $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0$?

A. The degree of f' is n. C. The constant term of f' is a_1.

B. The leading coefficient of f' is a_n. D. If $n = 1$, then $f'(x) = a_1 x + a_0$.

40. If $f(x) = \frac{x^2+2}{x+1}$, then $f'(x)$ is

A. $\frac{x^2+2x-2}{x+1}$ C. $\frac{-x^2-2x+2}{x+1}$

B. $\frac{x^2+2x-2}{(x+1)^2}$ D. $\frac{-x^2-2x+2}{(x+1)^2}$

41. If $f(x) = \frac{1}{\sqrt{x^3-5x+10}}$, then $f'(x)$ is

A. $\frac{1}{2\sqrt{x^3-5x+10}}$ C. $\frac{-1}{2\sqrt{x^3-5x+10}}$

B. $\frac{3x^2-5}{2\sqrt{x^3-5x+10}}$ D. $\frac{-3x^2+5}{2\sqrt{x^3-5x+10}}$

42. Assume that f is differentiable in the interior of an interval I. Which one of the following is true about f?

A. If $f'(x) > 0$ on I, then f is strictly decreasing on I.

B. If $f'(x) \leq 0$ on I, then f is increasing on I.

C. If $f'(x) = 0$ on I, then f is decreasing on I.

D. If $f'(x) > 0$ on I, then f is strictly increasing on I.

43. Which one of the following is true about the number c such that $f'(c) = 0$, where c is in the domain of f?

A. The tangent line at $(c, f(c))$ is vertical.

B. The number c is a critical number of f.

C. The function f is undefined at c.

D. The function f has value 0 at c.

44. Let c be a critical number of a function f. Then f has a relative minimum at c if

A. the sign of f' changes from positive to negative at c.

B. the sign of f changes from positive to negative at c.

C. the sign of f' changes from negative to positive at c.

D. the sign of f changes from negative to positive at c.

45. Which one of the following points is at where the function $f(x) = 3\sqrt{x+1}$ has a vertical tangent line?

A. $x = -1$ C. $x = 1$

B. $x = 0$ D. $x = 2$

46. What is the equation of the normal line of $f(x) = \frac{x}{x^2-1}$ at $x = 0$?

A. $y = -x$ C. $y = x$

B. $y = -x - 1$ D. $y = x + 1$

47. If the displacement of a particle at any time t is given according to the function $S(t) = t^3 - 2t^2$, where S is in meters and t is in seconds, then what is the acceleration of the particle at the end of $t = 2s$.

A. $6m/s^2$ C. $4m/s^2$

B. $0m/s^2$ D. $8m/s^2$

48. Given the total cost $C(x) = x^2+100$ and revenue $R(x) = x^3-x^2+30$ of a manufacturing company from the production and sale of x items. Which one of the following is true about the marginal analysis when 10 units are produced and sold?

A. The marginal cost is 200 birr.

B. The marginal revenue is 280 birr.

C. The marginal profit is 300 birr.

D. The marginal cost is 100 birr.

49. What is the Riemann sum of $f(x) = x^3 + 2$ on $[0, 5]$, where $[0, 5]$ is subdivided into 5 sub-intervals of equal width?

A. 235 C. 237

B. 110 D. 167

50. The Riemann sum of the areas of the rectangles under the graph of a non-negative function f over $[a, b]$ as the number of partitions of $[a, b]$ increases indefinitely is called

A. a definite integral. C. a derivative.

B. an indefinite integral. D. a continuity.

51. The value of $\int_{-2}^{1}(-2x + 5)dx$ equals

A. -2 C. 10

B. 18 D. -6

52. Which one of the following is true, where f and g are two functions?

A. $\int f(kx)dx = k\int f(x)dx$, where k is a constant

B. $\int(f + g)(x)dx = \int f(x)g'(x)dx + \int f'(x)g(x)dx$

C. $\int(f(x) - g(x))dx = \int f(x)dx - \int g(x)dx$

D. $\int f(x)g(x)dx = \int f(x)dx \int g(x)dx$

53. What is the area of the region under $f(x) = x^5 + 2$ over $[0, 2]$?

A. 80 C. 32

B. $\frac{38}{3}$ D. $\frac{44}{3}$

54. Which one of the following is the area of the region enclosed between the graph of $f(x) = x(x - 2)(x + 1)$ and the x-axis?

A. $\frac{-8}{3}$ C. $\frac{-9}{4}$

B. $\frac{37}{12}$ D. $\frac{5}{12}$

55. If the velocity of a traveling vehicle is given by $v(t) = 5t^4 - 10$ and the displacement at $t = 0$ is $s(0) = 5$, then what is $s(t)$?

A. $20t^3 + c$ C. $t^5 - 10t + 5$

B. $t^5 - 10t$ D. $t^5 - 5t + c$

56. The degree to which numerical data tends to spread about an average is called measures of

A. dispersion.
C. median.
B. central tendency.
D. mode.

57. A large value of range indicates that the data has

A. less dispersion.
C. less mean.
B. greater dispersion.
D. large mean.

58. Which one of the following is true about the range and the inter-quartile range?

A. Inter-quartile range considers the variability of each item in the data.
B. Inter-quartile range could possibly overcomes the limitations of the range.
C. Inter-quartile range ignores 50% between the first and the third quartiles.
D. Inter-quartile ranges of two data sets with the same range are the same.

Direction: For questions 30, 31, and 32, use the following data.

x	10	15	20	25	30
f	2	3	5	4	1

59. What is the mean deviation about the mean of the data?

A. 66.5
C. 19.7
B. 5
D. 4.44

60. What is the mean deviation about the median of the data?

A. 4.33
C. 15
B. 65
D. 1.33

61. What is the mean deviation about the mode of the data?

A. 20
C. 4.33
B. 3.73
D. 15

62. The standard deviation of the data $3, 6, 9, 12, 15$ is

A. 18
C. $3\sqrt{2}$
B. $2\sqrt{3}$
D. 9

63. What is the variance of the frequency table below?

x	0	1	2	3	4	5
f	3	3	1	2	2	1

A. $\frac{17}{6}$

B. $\frac{74}{6}$

C. $\frac{45}{6}$

D. 2

64. The variance of grouped frequency distribution below is

Class Interval	$1-5$	$6-10$	$11-15$	$16-20$	$21-25$
Frequency	6	6	4	2	2

A. 10

B. 41

C. 29.5

D. 14.75

65. The standard deviation of the grouped frequency distribution below is

Class Interval	$6-10$	$11-15$	$16-20$	$21-25$	$26-30$
Frequency	1	3	2	3	1

A. 18

B. $\sqrt{37}$

C. 35

D. $\sqrt{35}$