

ቀለሞች

t.me/keleme_2013

9. Use PMI to show that

$$n^2 \geq 2n + 1$$

For all $n \geq 3$

Proof: for $n=3$, $9 \geq 7$ ∴ it is true for $n=3$ } 0.5

Spse, it holds for $n=k$

$\Rightarrow k^2 \geq 2k+1$ } 0.5

for $n=k+1$,

$(k+1)^2 = k^2 + 2k + 1$

$\geq 2k+1+2k+1$ (by hypothesis)

$\geq 2k+1+1+1$ ($\because 2k \geq 1 \ \forall k \in \mathbb{Z}^+$

$= 2k+3$

$= 2(k+1)+1$

1.5 ⇒ it holds for $n=k+1$ also

∴ $n^2 \geq 2n+1$ (by PMI)

@keleme_2013

10. Find the solution set of the equation

$$z^{\frac{4}{5}} = -1$$

where z is a complex number.

Soln: $z^{\frac{4}{5}} = -1 \Rightarrow z^4 = (-1)^5 \Rightarrow z^4 = -1$ } 0.5

Then the 4th roots of -1 are

0.5 { $c_k = r^{1/4} e^{i\left(\frac{\theta + 2k\pi}{4}\right)}$,

0.5 { where $r=1$, $\theta = \pi$ and $k=0,1,2,3$

Then

1.5 {
$c_0 = e^{i\pi/4} = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$

$c_1 = e^{i\frac{3\pi}{4}} = \cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4} = -\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$

$c_2 = e^{i\frac{5\pi}{4}} = \cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4} = -\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}$

and $c_3 = e^{i\frac{7\pi}{4}} = \cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4} = \frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}$

Test Paper

Answer key

ADDIS ABABA SCIENCE AND TECHNOLOGY UNIVERSITY

COLLEGE OF NATURAL AND APPLIED SCIENCES

DEPARTMENT OF MPS

Test I(15%) Mathematics for Natural sciences (**Math 1011**)

Time allowed: 60 minutes. Date: Jun 25, 2024

Name: _____ Id No.:_____

Department: _____ Sec:_____

$i^{99} = i^{98} \cdot i = (i^2)^{49} i$
$= -i$

Part I: Short answer questions. (7 *pts.*)

1. The truth value of $(\exists x)(\exists y)(x, y \in \mathbb{Z})(4x + 3y = 45)$ is True
2. For any sets A, B and C, $A \cap B = A \cap C \Rightarrow B = C$. (True/False) False
3. If A is any set having **6** members, then $n(P(A)) =$ 64
4. For any set A, $\emptyset$ is both subset and member of $P(A)$. (True/False) True
5. The imaginary part of the complex number $z = (1 + i)^{20}$ is 0
6. The LUB of the set $S = \{x : \ln x < 2\}$ is e^2
7. The simplest form of $i^{99} =$ $-i$

Part II: Workout problems. (8 *pts.*)

8. Use truth table **only** to determine the validity of the following argument.

If the rain does not come, then the crop is ruined and the people will starve.
The crop is not ruined or the people will not starve.
Therefore, the rain comes.

Let p: The rain comes.
q: The crop is ruined.
r: The people will starve. } 0.5

Then the argument is
$\neg p \Rightarrow (q \wedge r), \neg q \vee \neg r \vdash p$ } 0.5

Let $p_1 = \neg p \Rightarrow (q \wedge r)$
$p_2 = \neg q \vee \neg r$
are premises

p	q	r	$\neg p$	$\neg q$	$\neg r$	p_1	$q \wedge r$	p_1	p_2
T	T	T	F	F	F		T	T	F
T	T	F	F	F	T		F	T	T
T	F	T	F	T	F		F	T	T
T	F	F	F	T	T		F	T	T
F	T	T	T	F	F		T	T	F
F	T	F	T	F	T		F	F	T
F	F	T	T	T	F		F	F	T
F	F	F	T	T	T		F	F	T

} 0.5

∴ It is valid } 0.5

0.5 } (∵ All the premises are true in the 2nd, 3rd and 4th rows and the conclusion p also true)